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Impact of operational subsidies for public transport on mode choice and spatial structure

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- Background
- Model
- Urban system equilibrium
- Sensitivity analysis
- Conclusion

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- **Background**
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Background

The acceleration of urbanization

- Serious environmental pollution, traffic congestion, traffic safety and other "urban diseases" have appeared in big cities.

Private car has risen dramatically

- By the end of 2019, the number of automobiles in China had reached 260 million, among which private cars had reached 207 million, exceeding 200 million for the first time.

Background

Transport policy

- **Congestion pricing**: It follows the Pigouvian tradition of charging for the external costs produced by an agent's decision.
- **Giving priority to public transport**: Public transport provides a mean of transporting more people using less space, thus diminishing congestion.
- **Public transport investment:** During the five years from 2012 to 2016, Beijing has invested 340 billion yuan (RMB) in the transportation sector, of which public transport investment accounts for 75% of the municipal transportation infrastructure investment.

Background

Literature review

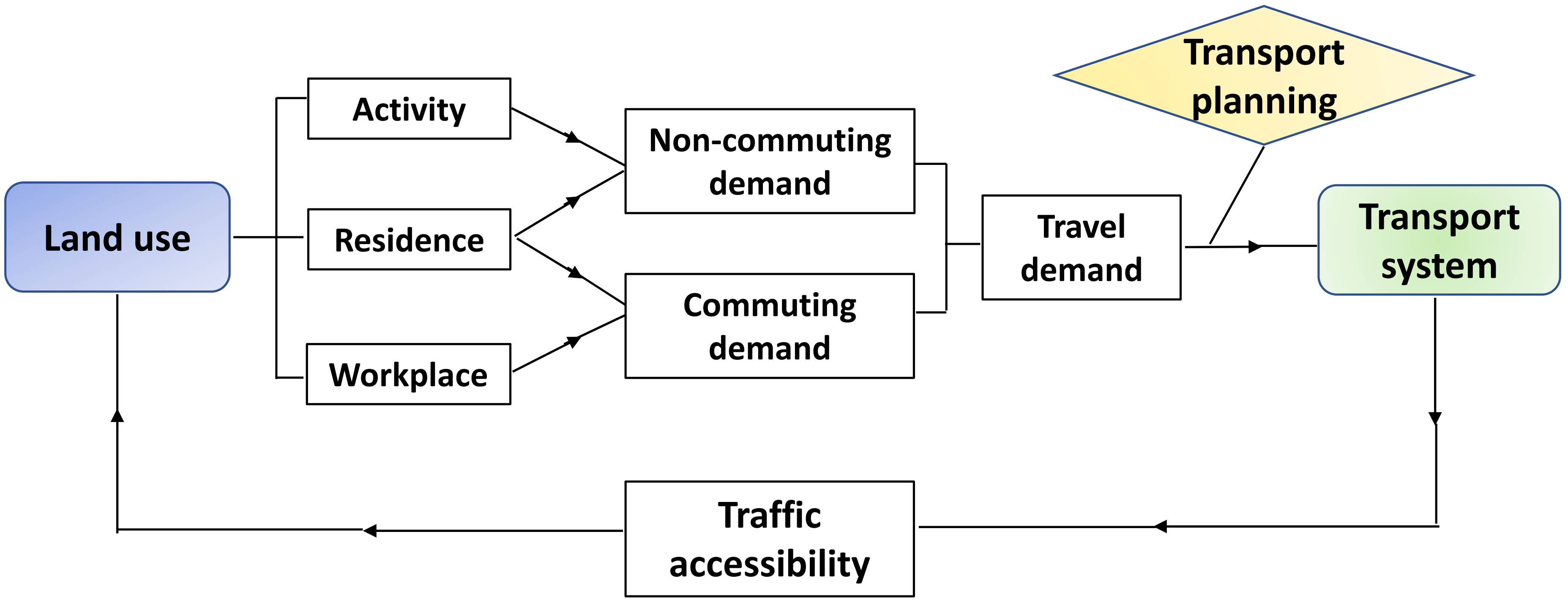
Topic	Considerations	Key references
Mode choice	Fare subsidy	Tisato, 1998; Shen,2020
	Fare subsidy and congestion pricing	Wu, 2011; Huang, 2004
	Fare subsidy and transit headway	Wang and Deng, 2019
Welfare and equity	Fare subsidy and city size	Asensio and Matas, 2014; **Börjesson,2019; 2020**
	Fare subsidy and bus frequency	Drevs, et al., 2014
	Fare subsidy	Basso and Silva, 2014

- **Operational bus strategies.** The study of public transportation subsidy should not only be limited to fare subsidy, but also consider the other operation and management strategies.

- **Spatial structure.** Most of the above studies focus on the choice of traffic mode, without considering the interaction between traffic system and urban spatial structure.

Background

The relationship between transport and land use



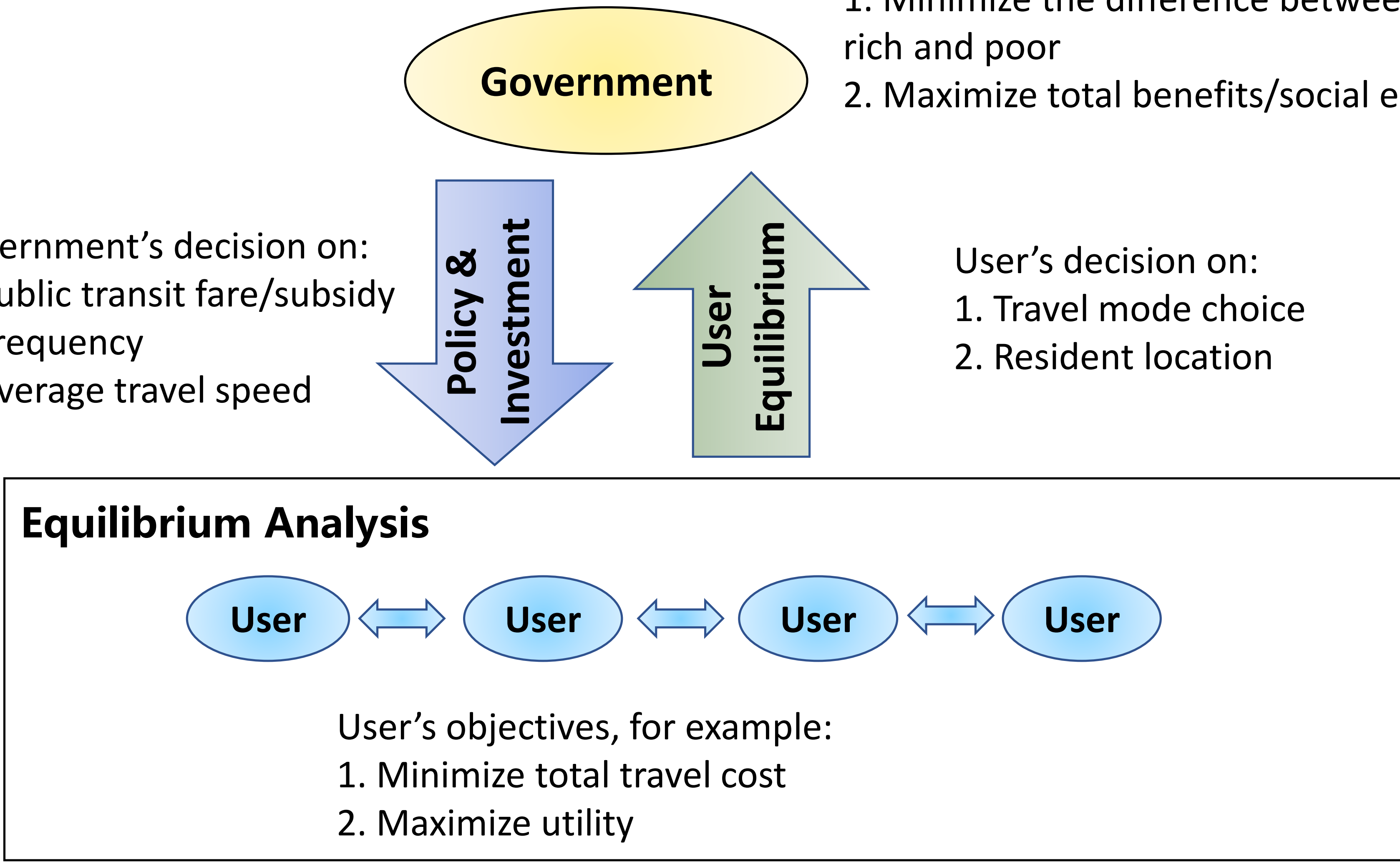
Background

Research Framework

Government's objectives, for example:
1. Minimize the difference between rich and poor
2. Maximize total benefits/social equity

Government's decision on:
1. Public transit fare/subsidy
2. Frequency
3. Average travel speed

User's decision on:
1. Travel mode choice
2. Resident location



User's objectives, for example:
1. Minimize total travel cost
2. Maximize utility

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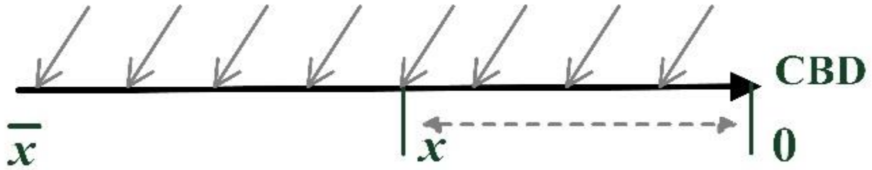
Model

Basic assumptions



- We consider a closed, monocentric city with a fixed total population.
- Residents are distributed in the linear corridor, commuting to work in CBD of the city with a mode choice of auto mode on the highway or public transport.
- All residents are assumed to be rational and heterogeneous with the different income level.
- The income will be spent on transportation, housing and other consumption.
- High-income people can travel either by car or by bus, while lower-income people can only travel by bus.
- Car use leads to road congestion, and public transport ignores the internal crowding.

Model

Mode choice

Travel cost for car users

capacity
automobile

$$TC_h^a = \beta_h \left(kN_h^a / s + x / v_a \right) + \tau_a.$$

number of car users
speed
money cost

Travel cost for bus users

bus

$$TC_i^b = \beta_i \left(w(f) + x / v_b \right) + \tau_b, \quad i = h, l.$$

high/low income
frequency

The critical point of spatial location for mode choice

$$x_m^* = \frac{\beta_h \left(kN_h^a / s - w(f) \right) + \left(\tau_a - \tau_b \right)}{\beta_h \left(1/v_b - 1/v_a \right)}.$$

For the high-income group, bus users will choose to live close to CBD, i.e., $x \le x_m^*$, car users will be away from CBD, i.e., $x > x_m^*$.

Model

Resident location

The utility of group *i* at location *x*

income

$$u_i = c_i = y_i - TC_i^j - R_i^j(x),\ i = h, l,\ j = a, b.$$

The rent function of mode *j* at location *x*

$$R_i^j(x) = y_i - TC_i^j - u_i,\ i = h, l,\ j = a, b.$$

The bid rent function at location *x*

$$R(x) = \max\{R_a, R_h(x), R_l(x)\}.$$

agricultural rent at city boundary

$$R_h(x) = \max\{R_h^a(x), R_h^b(x)\}$$

Assumption

$$\left|\frac{\partial R_h(x)}{\partial x}\right| = \frac{\beta_h}{v_b} > \left|\frac{\partial R_l(x)}{\partial x}\right| = \frac{\beta_l}{v_b}$$

(LeRoy and Sonstelie, 1983; de Bartolome and Ross, 2004; Glaeser et al., 2008)

- For high-income group, those who travel by bus will live closer to CBD than by car.
- For those who choose the same mode of transportation (bus), the high-income group will choose to live closer to the CBD than the low-income group.

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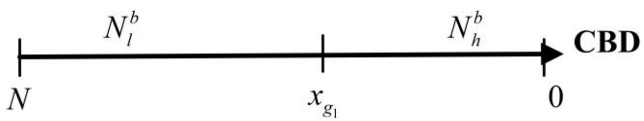
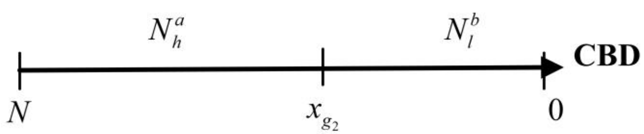
Model

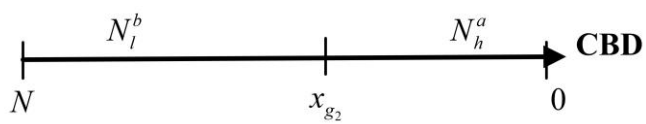
Urban structure (refer to LeRoy and Sonstelie, 1983 and Sasaki, 1990).

Case1. All rich commuters use public bus



Case2. All rich commuters use private car

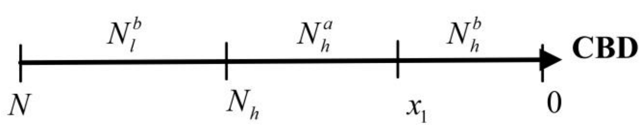




$$\left|\frac{\partial R_l(x)}{\partial x}\right| = \frac{\beta_l}{v_b} > \left|\frac{\partial R_h(x)}{\partial x}\right| = \frac{\beta_h}{v_a}.$$

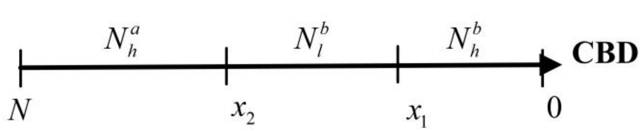
$$\beta_l / v_b < \beta_h / v_a .$$

Case3. Poor bus users live in the boundary (L+H)



$$\beta_l / v_b < \beta_h / v_a .$$

Case4. Poor bus users surrounded by rich commuters (HA+L+HB)



$$\beta_l / v_b > \beta_h / v_a .$$

Urban System equilibrium

Poor bus users live in the boundary (L+H)

- **Definition1.** We define the difference of utility between rich and poor, $\Delta u = u_h - u_l$ as the equity indicator, the greater the difference, the more unfair the strategy will be.

- **Definition2**. Feasibility refers to the fact that the government can get some corresponding profits to balance the cost after the implementation of a certain policy, for example, it can increase the aggregate land rent income(ALR) of the urban system, and we consider this policy to be feasible. Aggregate land rent income can be calculated by cumulative net rental income, i.e. $ALR = \int_0^{\bar{x}} \left(R(x) - R_a \right) dx$.

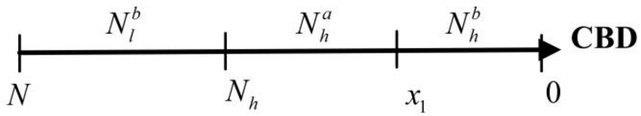
Urban System equilibrium

Poor bus users live in the boundary (L+H)

$$TC_h^a = \beta_h t_a\left(N_h^a, s, x\right) + \tau_a,$$

$$TC_i^b = \beta_i t_b\left(f, x\right) + \tau_b,\ \ i = h, l,$$

$\beta_l / v_b < \beta_h / v_a .$



Interior equilibrium conditions (Light, 2009; Nie and Liu, 2010)

$$\begin{cases} TC_h^a\left(N_h, s, N_h\right) > TC_h^b\left(f, N_h\right), \\ TC_h^b\left(f, N_h\right) > TC_h^a\left(0, s, N_h\right). \end{cases}$$

bus mode is more attractive if all travelers choose to drive

auto mode becomes more attractive if no one drives

Spatial equilibrium conditions of two groups

$$\begin{cases} R_l\left(\bar{x}\right) = R_a, \\ R_h^a\left(x_1\right) = R_h^b\left(x_1\right), \\ R_h^a\left(N_h\right) = R_l\left(N_h\right). \end{cases}$$

The cut-off point of mode choice for high income group

$$x_1 = x_m^* = \frac{\beta_h\left(kN_h^a/s - w\left(f\right)\right) + \left(\tau_a - \tau_b\right)}{\beta_h\left(1/v_b - 1/v_a\right)}$$

Urban System equilibrium

Poor bus users live in the boundary (L+H)

The number of people in the high income group who choose to travel by car

$$N_h^a = \frac{\left(\beta_h\left(w(f)+N_h/v_b\right)+\tau_b\right)-\left(\beta_h N_h/v_a+\tau_a\right)}{\beta_h\left[\left(1/v_b-1/v_a\right)+k/s\right]}.$$

Utility of two classes of inhabitants

$$\begin{cases} u_l = y_l - \beta_l\left(w(f)+N/v_b\right)-\tau_b-R_a, \\ u_h = y_h-\beta_l N_l/v_b-\tau_a-R_a-\Omega. \end{cases}$$

$$\Omega(v_b,\tau_b) \triangleq \frac{\left(1+s/(kv_a)\right)\beta_h N_h/v_b+\beta_h w(f)-\beta_h N_h s/\left(kv_a^2\right)-\tau_a+\tau_b}{1+\left(1/v_b-1/v_a\right)s/k}$$

The impact of these strategies

$$\partial u_i/\partial \tau_b<0, \partial u_i/\partial v_b<0, \partial u_l/\partial f>0, \partial u_h/\partial f=0.$$
$$\partial \Delta u/\partial \tau_b>0, \partial \Delta u/\partial f>0.$$

Urban System equilibrium

Poor bus users live in the boundary (L+H)

Inputs

Parameter	Value	Parameter	Value
Vehicle congestion k	0.6	Value of time for high-income group β_h (RMB/h)	14
Road capacity s (veh/h)	5400	Value of time for low-income group β_l (RMB/h)	10
Money cost of car τ_a (RMB)	10	Average annual income for the rich y_h (RMB/year)	80000
Money cost of bus τ_b (RMB)	5	Average annual income for the rich y_l (RMB/year)	60000
Frequency of bus f (run/h)	1.2	The number of low-income group N_l	15000
Speed of bus v_b (km/h)	40(L+H) 30(HA+L+HB)	The number of low-income group N_l	15000
Speed of car v_a (km/h)	50	Agricultural rent at the city boundary R_a (RMB/year)	30000

Poor bus users live in the boundary (L+H)

Strategies change		u_h	u_l	Δu
v_b	40 km/h	42150.96	21666.67	20484.29
	42 km/h	42249.78	21845.24	20404.54
	44 km/h	42339.78	22007.57	20332.21
τ_b	7 RMB	41208.77	20666.67	20542.10
	6 RMB	41679.86	21166.66	20513.20
	5 RMB	42150.96	21666.67	20484.29
f	1.0 run/h	41601.35	21250.00	20351.35
	1.2 run/h	42150.96	21666.66	20484.29
	1.4 run/h	42543.53	21964.29	20579.25

Proposition 1. When low-income groups live in urban boundaries, improving the quality of bus service can increase the utility of all groups. However, the strategy of increasing the frequency of bus should be cautious, which may exacerbate the inequality of social groups.

Urban System equilibrium

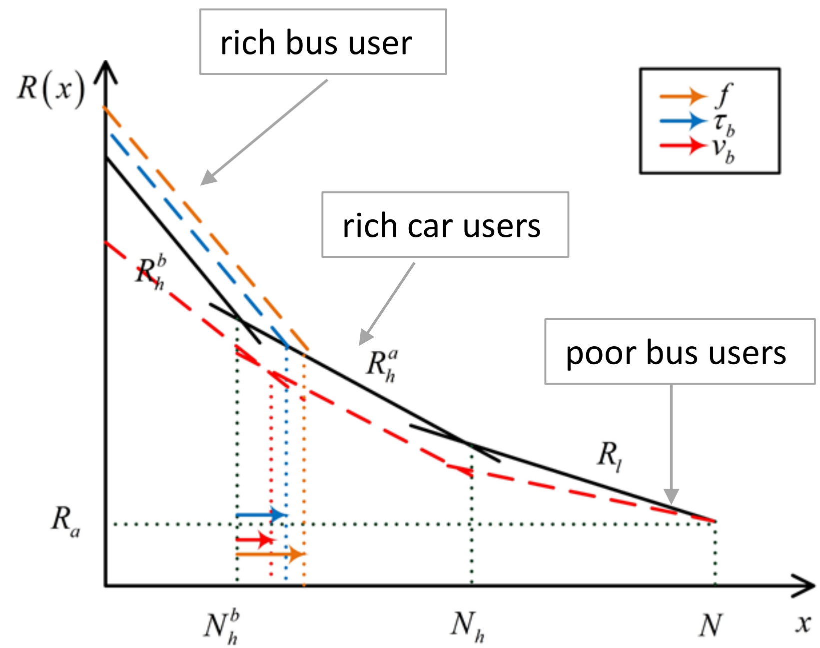
Poor bus users live in the boundary (L+H)

The rent function at spatial equilibrium

$$R(x)=\begin{cases} R_h^b(x)=\dfrac{\beta_l N_l}{v_b}-\beta_h\left(w(f)+\dfrac{x}{v_b}\right)+\tau_a-\tau_b+\Omega+R_a,\ x\le x_m^*, \\ R_h^a(x)=\dfrac{\beta_l N_l}{v_b}+\dfrac{\beta_h(N_h-x)}{v_a}+R_a, x_m^*<x\le N_h, \\ R_l(x)=\dfrac{\beta_l}{v_b}(N-x)+R_a, x>N_h. \end{cases}$$

Proposition 2.

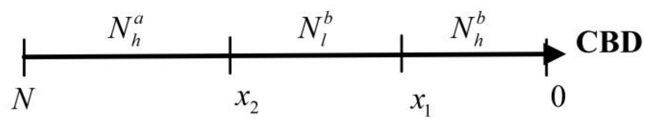
- When low-income groups live in urban boundaries, it is a feasible measure to increase the frequency or decrease the fare of bus.
- Only the fare subsidy is the feasible and fair strategy.



Urban System equilibrium

Poor bus users surrounded by rich commuters (HA+L+HB)

$\beta_l / v_b > \beta_h / v_a$.



$$TC_h^a = \beta_h t_a\left(N_h^a, s, x\right) + \tau_a,$$

$$TC_i^b = \beta_i t_b\left(f, x\right) + \tau_b, \quad i = h, l,$$

Spatial equilibrium conditions of two groups

$$\begin{cases} R_h^a\left(\bar{x}\right) = R_a, \\ R_h^b\left(x_1\right) = R_l\left(x_1\right), \\ R_h^a\left(x_2\right) = R_l\left(x_2\right). \end{cases}$$

The cut-off point of pattern choice of high income group

$$x_1 = v_b\left[\frac{\left(y_h - y_l\right) - \left(u_h - u_l\right)}{\beta_h - \beta_l} - w\left(f\right)\right].$$

Urban System equilibrium

Poor bus users surrounded by rich commuters (HA+L+HB)

The number of people travelling by car

$$\Gamma = \beta_h \left(\frac{kN_h}{s} + \frac{N_l}{v_a} \right) - \frac{\beta_l N_l}{v_b} - \beta_h v_b w(f) \left(\frac{1}{v_a} - \frac{k}{s} \right)$$

$$N_h^a = N_h - v_b \left(\frac{(\tau_a - \tau_b) + \Gamma}{(1-\Psi)(\beta_h - \beta_l)} - w(f) \right),$$

$$\Psi = \frac{\beta_h v_b (1/v_a - k/s) - \beta_l}{\beta_h - \beta_l}$$

Utility of two classes of inhabitants

$$\begin{cases} u_h = y_h - \tau_a - R_a - \beta_h \left(\dfrac{kN_h}{s} + \dfrac{N}{v_a} \right) + \beta_h v_b \dfrac{k}{s} \left(\dfrac{(\tau_a - \tau_b) + \Gamma}{(1-\Psi)(\beta_h - \beta_l)} - w(f) \right), \\ u_l = y_l - \tau_a - R_a - \beta_h \left(\dfrac{kN_h}{s} + \dfrac{N}{v_a} \right) + \left(\dfrac{\beta_h v_b k/s}{\beta_h - \beta_l} + 1 \right) \left(\dfrac{\tau_a - \tau_b + \Gamma}{1-\Psi} \right) - \beta_h v_b w(f) \dfrac{k}{s}. \end{cases}$$

The impact of these strategies

$$\partial u_i / \partial \tau_b < 0, \partial u_i / \partial v_b > 0, \partial u_i / \partial f > 0.$$

$$\partial \Delta u / \partial \tau_b > 0, \partial \Delta u / \partial f > 0, \partial \Delta u / \partial v_b > 0.$$

Urban System equilibrium

Poor bus users surrounded by rich commuters (HA+L+HB)

Strategies change		u_h	u_l	Δu
v_b	30 km/h	37518.75	19487.50	18031.25
	32 km/h	38597.96	20358.51	18239.45
	34 km/h	39407.19	21017.54	18389.65
τ_b	7 RMB	37927.98	19515.14	18412.84
	6 RMB	38374.40	19999.83	18374.57
	5 RMB	38820.83	20484.52	18336.31
f	1.0 run/h	38300.00	20085.71	18214.29
	1.2 run/h	38820.83	20484.52	18336.31
	1.4 run/h	39192.86	20769.39	18423.47

Proposition 3. When low-income groups are surrounded by high-income groups, improving the service quality of buses can increase the utility of all residents. But only the fare subsidy strategy is conducive to the realization of urban equity measures.

Urban System equilibrium

Poor bus users surrounded by rich commuters (HA+L+HB)

The rent function at spatial equilibrium

$$R(x)=\begin{cases} R_h^b(x)=-\beta_h\left(w(f)+\dfrac{x}{v_b}\right)+y_h-\tau_b-u_h,\ x\le x_1, \\ R_l(x)=-\beta_l\left(w(f)+\dfrac{x}{v_b}\right)+y_l-\tau_b-u_l, x_1< x\le x_2, \\ \boxed{R_h^a(x)=\dfrac{\beta_h}{v_a}(N-x)+R_a, x>x_2.} \end{cases}$$

→ unrelated to the bus operation strategies

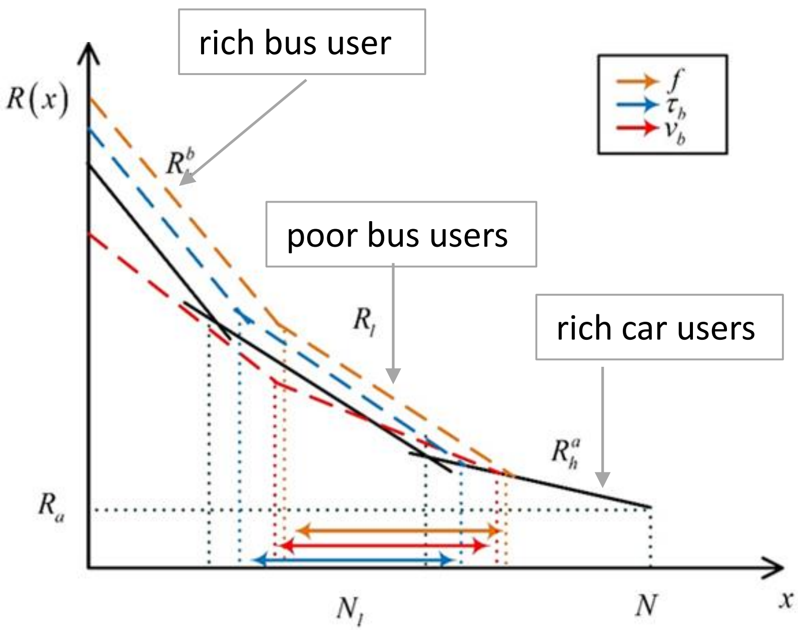
The change of bus service level will not affect the rent function of car users in the city boundary, but will change the rent function curve of bus users.

Urban System equilibrium

Poor bus users surrounded by rich commuters (HA+L+HB)

location	rent	strategy	slope
$x \le x_1$	$R(x)=R_h^b(x)$	τ_b ↓	→
		f ↑	→
		v_b ↑	↓
$x_1 < x \le x_2$	$R(x)=R_l(x)$	τ_b ↓	→
		f ↑	→
		v_b ↑	↓
$x > x_2$	$R(x)=R_h^a(x)$	τ_b ↓	→
		f ↑	→
		v_b ↑	→

Proposition 4: When low-income groups are surrounded by high-income groups, reducing bus fares is a measure that combines feasibility with equity.



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Sensitivity analysis

The number of high-income population

In a closed monocentric city, when the distribution of residents obeying the 'L+H' pattern, the increase in the number of high-income groups leads to:

(a) The utility level of high-income groups decreases, while the utility level of low-income groups remains unchanged, and the city tends to be fair;

(b) An increase in car use;

(c) Rental levels have risen in central areas and have generally fallen for low-income groups near urban boundaries.

$$\frac{\partial u_h}{\partial N_h} = \frac{(\beta_h - \beta_l)/v_b + (1/v_b - 1/v_a)(\beta_l/v_b - \beta_h/v_a)s/k}{1-(1/v_b - 1/v_a)k/s} < 0,\ \frac{\partial u_l}{\partial N_h} = 0,\ \frac{\partial \Delta u}{\partial N_h} < 0.$$

$$\frac{\partial N_h^a}{\partial N_h} = \frac{1/v_b - 1/v_a}{1/v_b - 1/v_a + k/s} > 0.$$

$$\frac{\partial R(0)}{\partial N_h} = -\frac{\partial u_h}{\partial N_h} > 0,\ \frac{\partial R(x_2)}{\partial N_h} = -\frac{\beta_l}{v_b} < 0.$$

Sensitivity analysis

The number of high-income population

In a closed monocentric city, when the distribution of residents obeying the pattern of 'HA+L+HB', the increase in the number of high-income groups leads to:

(a) The utility level of high-income groups decreases, and the low-income groups' utility increases, and the city tends to be fair;

(b) The number of people using cars has increased;

(c) The rent level in central areas and near city boundaries (for the rich) has increased, while the rent level of low-income groups has decreased as a whole.

$$\frac{\partial u_h}{\partial N_h}=\frac{\beta_l-\beta_h}{1-v_b\left(1/v_a-k/s\right)}<0,\ \frac{\partial u_l}{\partial N_h}=\frac{\left(\beta_h-\beta_l\right)\left(\beta_h k/s+\beta_l/v_b-\beta_h/v_a\right)}{\beta_h\left(1-v_b\left(1/v_a-k/s\right)\right)}>0,\ \frac{\partial \Delta u}{\partial N_h}<0.$$

$$\frac{\partial N_h^a}{\partial N_h}=1-\frac{v_b k/s}{1-v_b/v_a+v_b k/s}>0.$$

$$\frac{\partial R(0)}{\partial N_h}=-\frac{\partial u_h}{\partial N_h}>0,\ \frac{\partial R(x_1)}{\partial N_h}=-\frac{\beta_l}{v_b}\frac{\partial x_1}{\partial N_h}-\frac{\partial u_l}{\partial N_h}<0,\ \frac{\partial R(x_2)}{\partial N_h}=-\frac{\beta_h}{v_a}\frac{\partial x_2}{\partial N_h}>0.$$

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Conclusion

Summary

- *The influence of public transport operating subsidies on the mode choice and urban spatial structures has been studied.*
- *Analyze the impact of the change in the number of high-income groups on the urban system under each urban structure.*

Implications for transportation agencies

- *This research contributes to the operational management of bus system and provides a useful guideline with regard to government subsidy policy.*
- *In addition, the government should adjust the operational subsidy policy in different cities according to the different spatial structure or population.*

Further research

- *Relax the assumption of one unit of land use for each person, and consider the heterogeneity of housing consumption;*
- *Consider the capacity of the bus or the in-vehicle uncomfortable congestion cost.*

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Thanks for listening!

Looking forward to your comments and advice

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